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THE DESIGN OF NAILED JOINTS FOR CONTINUOUS TIMBER BEAMS [1]

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The design of light timber structures for farm service buildings or dwellings presents an opportunity for conserving materials by using continuous members. Immediate application of the theory of continuous members to the design of roof purlins (often 2 by 4's on edge) on pole barns, upper chord members of trussed rafters, and floor joists for houses or farm buildings could reduce the required size of member so designed.

A planned series of lap-nailed joints were tested under bending and bending-shear load combinations. These tests verified the accuracy of the general design formulas and established the parameters necessary for successful rotational-resistant nailed joint design. Figure 1 shows general views of the testing equipment. The tests are briefly summarized and the design procedures for practicing engineers are presented here. The reader is referred to references (2)[2] and (6) for the design of plywood haunches and side-plate joints, respectively.

Figure 1.--General view of testing equipment: A, for applying and measuring load; B, for measuring deflection and joint rotation.

1/ In cooperation with the Virginia Agricultural Experiment Station.
2/ See Selected References at end of this article.

REVIEW OF RESEARCH BASIS FOR JOINT DESIGN

The study of the design for nailed joints involved more than 200 individual tests and included both sixteenpenny common nails and hardened screw nails. All joints tested were lapped 2-inch members of Douglas-fir. The evidence gained from these tests can be extended to the design of joints with other size nails and 1-inch lumber.

This investigation and the results are reported by Kent and others (3). In brief, the variables investigated were the elastic joint modulus (JM_e) and the ultimate joint modulus (JM_u), with two types of nails, and the resulting joint rotation under different values of moment.

It was concluded that all nailed joints rotate under moment; however, for joints designed to be as strong as the connected members, the increased stiffness due to doubling the members across a lapped joint resulted in the measured deflection of joined members approximately equaling the deflection of a continuous member. Furthermore, the study revealed that joints may be designed by either elastic methods or ultimate methods.

HOW TO DESIGN NAILED JOINTS

Ultimate Design

The concept of ultimate design has had little application to timber structures. Ultimate design requires visualizing the deformations that occur just prior to failure.

The moment-resisting properties of the joint are the forces on each nail multiplied by the distance from each nail to the nail pattern centroid. In a nailed joint, one would expect that joint rotation would be very large near failure, and it is apparent that each nail would have yielded so that it would be loaded at or near its ultimate strength. That is, at loads approaching failure, the load on each nail approximates that on every other nail due to plastic deformation. Note that the assumption presumes near failure and is more nearly correct at ultimate nail loads than it is for safe loads. The sum of the distances of each nail from the nail pattern centroid multiplied by the ultimate strength of the nail is a measure of the ultimate strength of the nailed joint.

For example, notice that the distances from the nail pattern centroid to each nail are labeled r_i (figure 2) where i is an integer ranging from 1 to n--the number of nails. The summation of all lever arms, 1 through n, is a parameter of the ultimate strength of this joint. If we define ultimate moment (M_u) as the ultimate nail load (F_u) multiplied by the total length of rotational lever arms, or $F_u \sum_{1}^{n} r_i$, then F_a, the design nail load, is directly proportional to the ultimate nail load by a factor of safety. The design strength of the joint against rotation can be obtained by dividing the ultimate moment by the same factor.

The design moment equals the design nail load multiplied by the summation from 1 to n of r_i's. Thus, by applying the same safety factor to both sides of the ultimate relation, we have obtained a design relation that includes the recommended design load value for the nail, which may be obtained from curves or tables in published specifications (5) and from the physical properties of the joint.

These relations are outlined mathematically as follows:

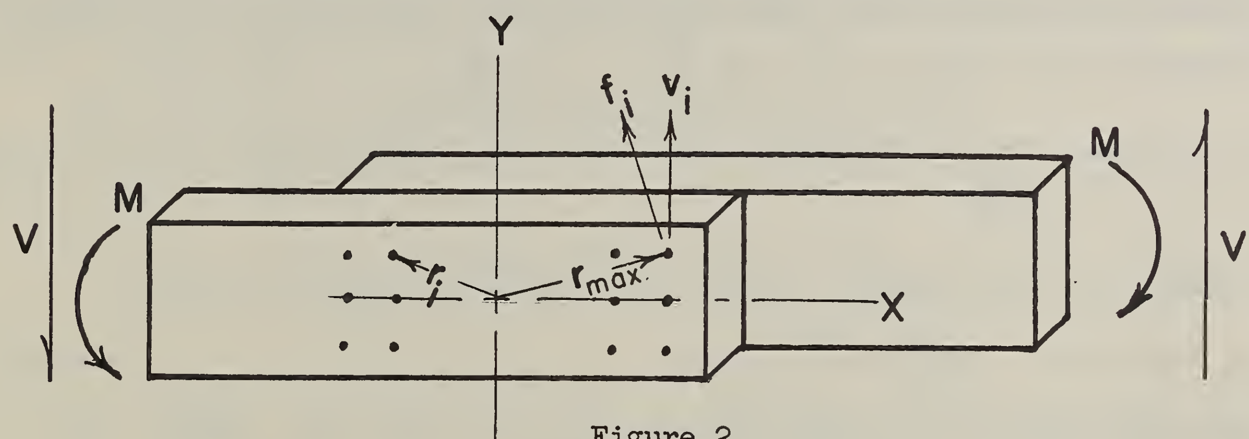

Figure 2

The design nail load (F_a) is defined as $F_a = \frac{F_u}{FS}$, where FS is the factor of safety. M_a, the allowable moment, may be defined as $\frac{M_u}{FS}$ or $M_a = F_a \sum_{1}^{n} r_i$. Since r_i may be resolved into components so $r_i = (x_i^2 + y_i^2)^{1/2}$ and the ultimate joint modulus is defined as $JM_u = \sum_{1}^{n} r_i = \sum_{i}^{n} (x_i^2 + y_i^2)^{1/2}$.

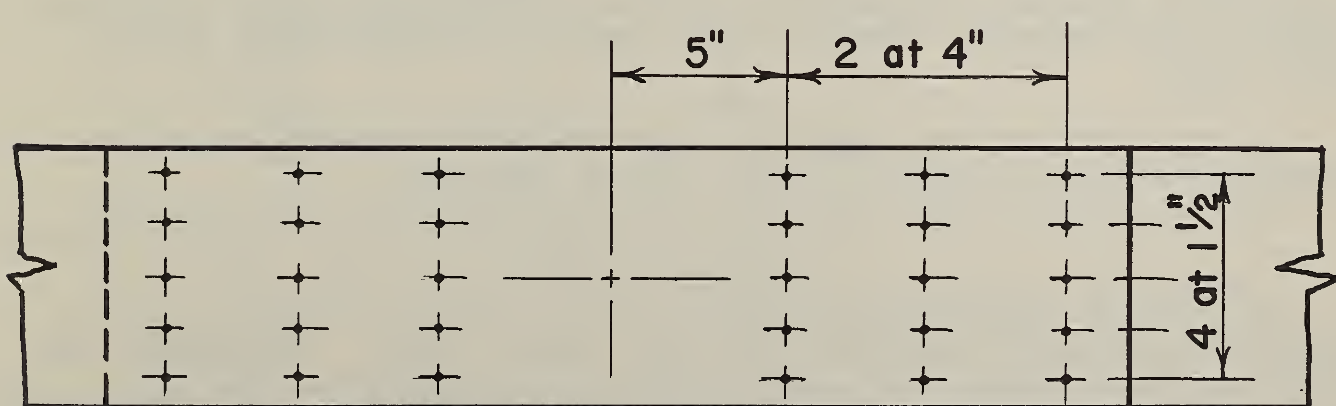

Figure 3

Example of Ultimate Design

Suppose figure 3 represents a joint of two 2 by 8's. The joint should be designed to be as strong as the connected members. For the 2 by 8, M_a, the allowable moment, is the allowable fiber stress times the section modulus and should equal the allowable moment of the nails (M_a), or the allowable nail load times the summation of the moment arms. The section modulus of a 2 by 8 is 15.2.

Assume an extreme fiber stress of 1,250 p.s.i., then $M_a = 1{,}250 \times 15.2$. The strength of the joint is $M_a = F_a \sum_i^n r_i$, or $F_a \sum_i^n r_i = 1{,}250 \times 15.2$.

If it is assumed that sixteenpenny nails are used, the allowable load per nail (5) is $F_a = 109 \dfrac{1.625}{2.33} = 76$ lb.

Solving for the required ultimate joint modulus ($\sum r_i$) we obtain,

$$\sum r_i \text{ required} = \frac{1{,}250 \times 15.2}{76} = 251$$

Checking the above joint, we find,

$$\sum r_i = \sum (x_i^2 + y_i^2)^{1/2} = 4 \Big[(5^2 + 3^2)^{1/2} + (9^2 + 3^2)^{1/2} + (13^2 + 3^2)^{1/2} + (5^2 + 1.5^2)^{1/2} + (9^2 + 1.5^2)^{1/2} + (13^2 + 1.5^2)^{1/2} \Big] + 2\ (5 + 9 + 13) = 278.$$

This joint is amply strong, since the ultimate joint modulus of 278 is greater than the required JM_u of 251. Similarly, the design moment exceeds the strength of the member in bending and the extreme nail load $F = \dfrac{M}{JM_u} = 68.5$ is obviously less than the allowable nail load of 76. Any one of these three criteria (joint modulus, moment, or nail load) can be compared to indicate joint safety.

A simplified estimate could have been made by multiplying 30 nails by the average distance of 9 inches, giving a JM_u of about 270.

Elastic Design

In contrast with ultimate analysis, elastic analysis requires that one visualize load-strain relations within the elastic region. If a linear load-slip relation is assumed for nails in wood, it is evident that the load carried by individual nails in the nail pattern (figure 2) are directly proportional to the distance of the nail from the rotational center of the pattern.

Defining the nail farthest from the centroid as the extreme nail, and the force on the extreme nail as F_{max}, the load on each nail is related to F_{max} by the ratio $\frac{r_i}{r_{max}}$. As before, the summation of the forces on each nail multiplied by the distance of the nails from their rotational centers is equal to the resisting moment of the joint pattern. Therefore, these two relations--that of the force on each nail to the maximum force and that of all the forces and lever arms to the resisting moment--determine that the resisting moment is equal to F_{max} divided by r_{max} multiplied by the summation of the square of all the r's.

This relation is expressed mathematically as follows:

Refer again to figure 2:

$$M = F_1 r_1 + F_2 r_2 + + + F_n r_n, \text{ but } F_i = F_{max} \left(\frac{r_i}{r_{max}}\right), \text{ or } F_i = \frac{F_{max}}{r_{max}} (r_i)$$

Therefore,

$$M = \frac{F_{max}}{r_{max}} (r_i^2 + r_2^2 + + + r_n^2), \text{ or } M = \frac{F_{max}}{r_{max}} \sum_1^n r_i^2, \text{ where } \sum_1^n r_i^2 / r_{max}$$

is defined as the elastic joint modulus (JM_e) and $JM_e = \dfrac{\Sigma x_i^2 + \Sigma y_i^2}{(X_{max}^2 + Y_{max}^2)^{1/2}}$

Example of Elastic Design

Refer to figure 3:

$$\frac{\Sigma r_i^2}{r_{max}} = \frac{\Sigma x_i^2 + \Sigma y_i^2}{(X_{max}^2 + Y_{max}^2)^{1/2}} = \left[10\,(5)^2 + 10\,(9)^2 + 10\,(13)^2 + 12\,(3)^2 + 12\,(1.5)^2\right] \Big/ \left[(13)^2 + (3)^2\right]^{1/2} = 2{,}883/13.35$$

Therefore, the allowable moment for this joint when checked from the elastic standpoint is as follows:

$M_a = F_a \, JM_e$, where $F_a = 76$ (by procedures given in previous example)

$= \dfrac{76 \times 2{,}883}{13.35} = 16{,}400$ in.-lb. $<$ 19,000 in.-lb. allowable for the member.

This joint is not sufficiently strong when analyzed elastically; whereas, it was found to be amply strong when ultimate strength procedures were used.

Therefore, when designing by the elastic method, it is necessary to redesign the joint to obtain a larger joint modulus. Let us begin by spacing each nail group 2 inches farther from the pattern centroid (figure 4).

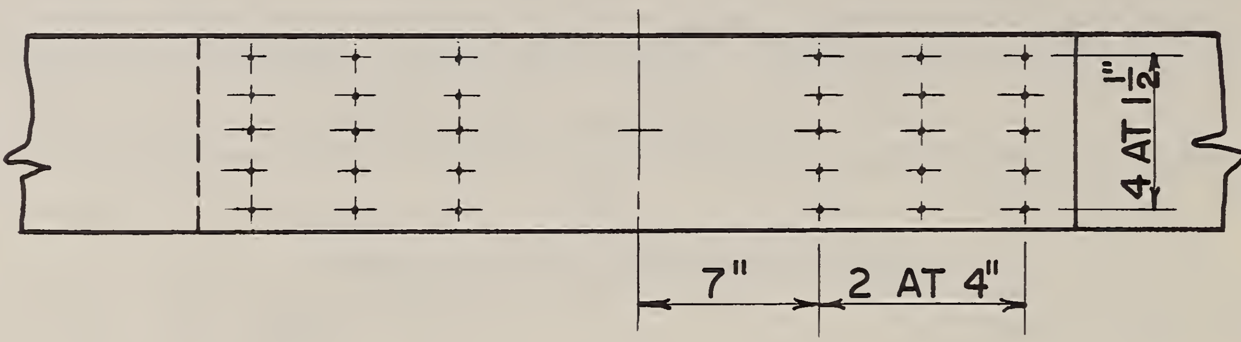

Figure 4

$$\Sigma r_i^2 = \Sigma x^2 + \Sigma y^2 = 10\,(7)^2 + 10\,(11)^2 + 10\,(15)^2 + 12\,(3)^2$$

$$+ 12\,(1.5)^2 = 4{,}085 \quad \text{and} \quad \frac{\Sigma r_i^2}{r_{max}} = \frac{4{,}085}{15.3} = 267.$$

$\dfrac{\Sigma r_i^2}{r_{max}}\, F_a = 267 \times 76 = 20{,}300 \quad 1{,}250 \times 15.2$, therefore, this joint is acceptable.

Comparison of Ultimate and Elastic Design

Experimentally, joint elasticity followed conservative design (elastic principles) for intermediate loads, but gradually became plastic and approached radical design (ultimate principles) as the joint was heavily loaded.

In general, members designed for stiffness (where deflection controls) should be joined, using the modulus of joint elasticity (JM_e). In those structures having strength or resistance to failure as the main design consideration, the ultimate modulus (JM_u) is satisfactory.

The maximum theoretical ratio of $\dfrac{JM_u}{JM_e}$ is 3/2 for a rectangular nail pattern, because this ratio is

$$\frac{r_1 + r_2 + r_3 \ldots\ldots\ldots\ldots r_n}{r_1^2 + r_2^2 + r_3^2 + \ldots\ldots r_n^2/r_n} \quad \text{or} \quad \frac{r_n\,(r_1 + r_2 + r_3 \ldots\ldots\ldots r_n)}{r_1^2 + r_2^2 + r_3^2 \ldots\ldots\ldots\ldots r_n^2}$$

By the theory of numbers, permitting a 1 to 1 correspondence of the radii r_1, r_2, etc., the numerator is $n\left[\frac{n(n+1)}{2}\right]$ and the denominator is $\frac{n(n+1)(2n+1)}{6}$ so the ratio is $\frac{3n}{2n+1}$.

L'Hospital's rule states that the first derivative of the numerator over the first derivative of the denominator is the limit of the ratio as the variable becomes infinite. Hence:

$$\lim_{n \to \infty} \frac{3n}{2n+1} = 3/2$$

Practically the ratio varies between 1.1 and 1.3 for most joints.

Design Against Shear-Moment Combinations

Refer to figure 2 and note that the force on the nail is not limited to rotational forces normal to the rotational radius but also includes a shear component parallel to the applied load. For symmetrical joints two of the extreme nails will be subjected to the maximum value of these two forces, and it remains for us to insure that this does not exceed the permissible force on the nail.

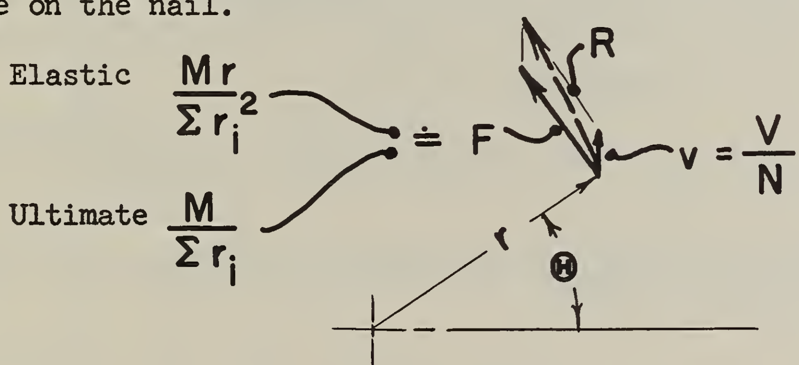

Figure 5

A vector diagram of forces on every nail in a pattern subjected to both rotational and shearing forces is shown in figure 5. The total force may be computed by the law of cosines; i.e.,

$$R = (v^2 + F^2 + 2vF\cos\theta)^{1/2},$$

where R is the resultant vector acting on the nail due to F, nail force caused by rotational moment, and v, the average shear force on each nail, which is assumed to be equal for all nails (4) and is determined by dividing the value of shear on the member at the joint by the number of nails in the joint. For small angles between the horizontal and r_{max}, $\cos\theta \approx 1$, and $R_{max} \approx F_{max} + V_{max}$. It will be helpful in synthesizing a joint for design purposes to note that the magnitude of v is solely a function of the number of nails while F is a function of the shape of the nail pattern.

As previously stated, the joint should always be designed for the full bending strength of the member. If, however, the joint is not located at a point of maximum moment, the extreme nail will not be loaded to capacity and the joint may be amply strong in shear. If it is not, the simplest solution is the addition of nails near the rotational center to reduce the value of the shear load on the extreme nail.

Example Problem with Moment and Shear

Consider the following moment-shear diagram for a 2 by 4 roof purlin joined over a roof truss (figure 6).

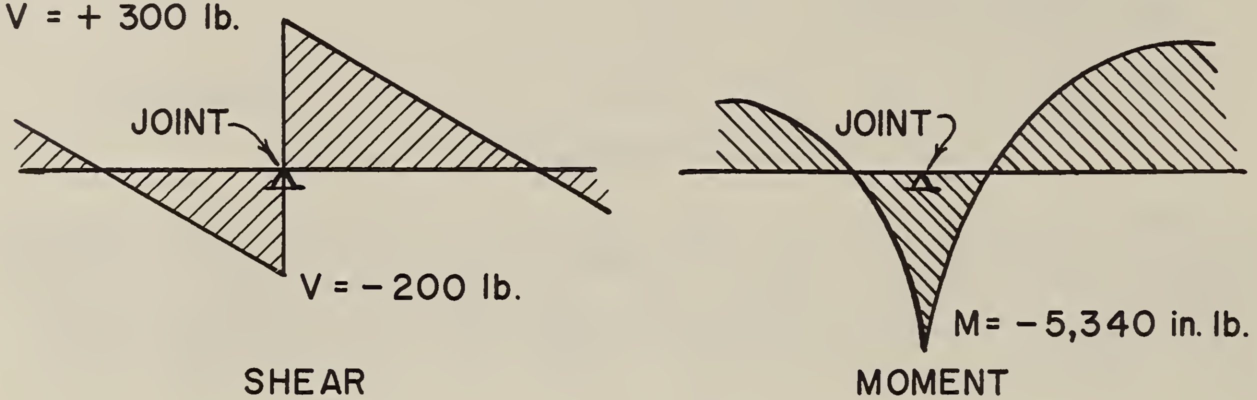

Figure 6

The joint should be designed for the maximum moment, 5,340 in.-lb., and the maximum shear, 300 lb.

For a first approximation we selected the pattern in figure 7 of a joint for 2 by 4's and designed it elastically.

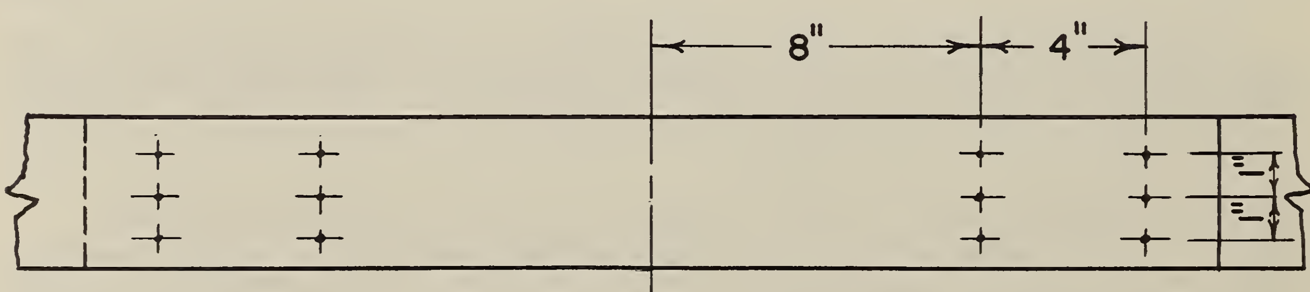

Figure 7

$$JM_e = \frac{\Sigma r_i^2}{r_{max}} = \frac{6\ (8)^2 + 6\ (12)^2 + 8\ (1)^2}{\left[(12)^2 + (1)^2\right]^{1/2}} = 104.7$$

$$F_{max} = \frac{5{,}340}{104.7} = 51; \quad v_{max} = \frac{300}{12} = 25; \quad \cos\Theta = \frac{12}{(145)^{1/2}} \approx 1.$$

$R_{max} = F_{max} + V_{max} = 51 + 25 = 76$ (allow for sixteenpenny nails) and this joint is sufficiently strong.

Suppose the joint had not been strong enough. There would be two alternatives: (1) Increase the number of nails in the area near the centroid, which would reduce both F_i and v; or (2) Spread the same number of nails father apart, which would reduce F but require more lumber in the joint.

The interaction of maximum moment and shear combinations has been plotted in figure 8 for three different 2 by 4 joints using sixteenpenny nails. This figure relates the behavior of three joints of nearly equal elastic joint moduli but with different numbers of nails. The different slopes of the relation for the three joints show that the permissible moment decreases rapidly as shear is added to joints of few nails. In contrast, as the number of nails is increased from 6 in joint A to 18 in joint C the permissible moment is greatly increased for large values of shear. Thus, the figure graphically illustrates that elastic modulus is primarily a function of nail pattern and that shear strength is a function of the total number of nails.

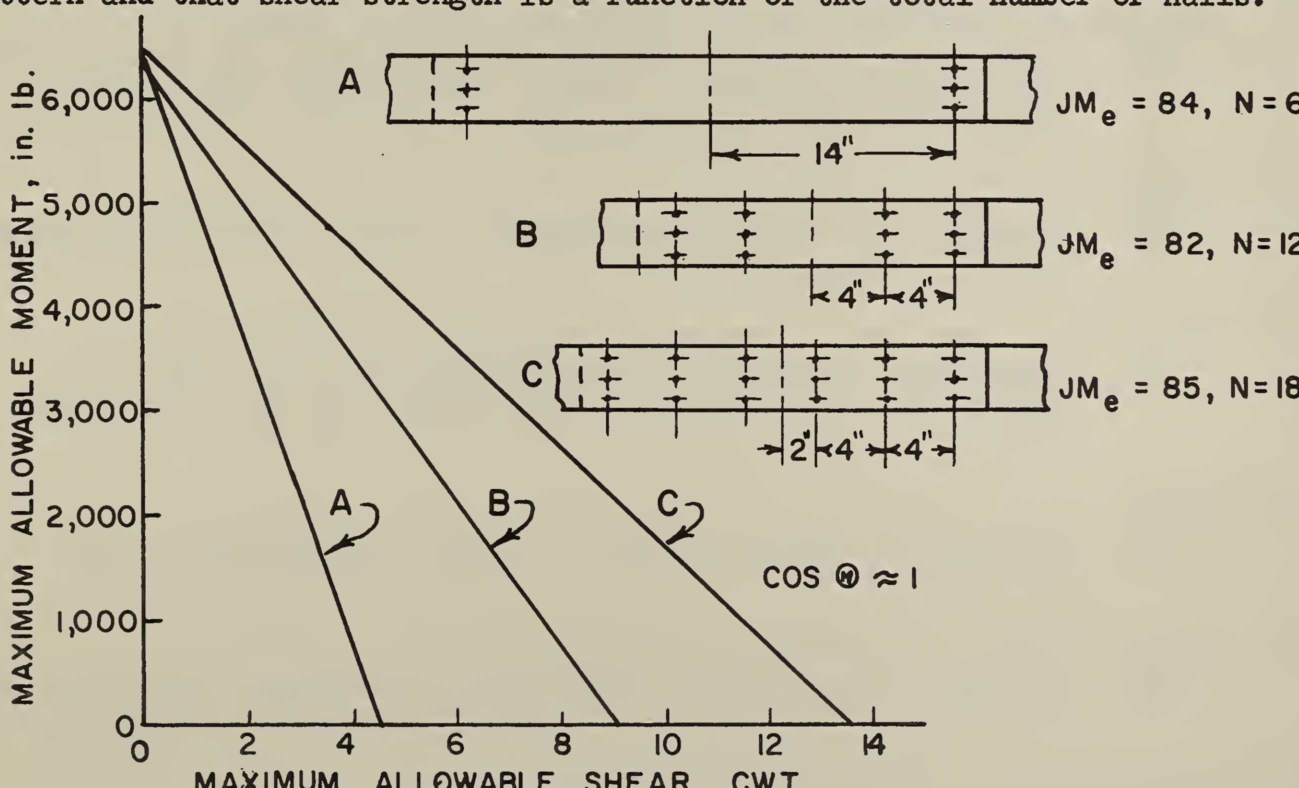

Figure 8

SUMMARY AND CONCLUSIONS

Two methods for the design of nailed joints are presented: (1) Ultimate design and (2) elastic design. Use of either of these methods will result in safe, dependable nailed joints, provided one follows all previously established rules for allowable nail loads and nail spacings from edge or on center. The elastic analysis is somewhat more conservative than the ultimate analysis. The theoretical maximum ratio of allowable moment by ultimate-strength analysis to that obtained by the elastic method is one and one-half for rectangular nail patterns; however, this ratio will usually range between 1.1 and 1.3. Procedures for calculating the ultimate joint modulus appear simpler; however, this is deceptive. The elastic modulus actually requires less time to calculate. In either case, it is recommended that the joint be designed to be as strong in bending as the member itself, rather than designing the joint for the moment at the joint. This will result in apparent continuity throughout the member and simplify greatly the calculation for deflection. A close approximation for deflections may be obtained by assuming that the joined member is a continuous beam of constant cross-section throughout its length. Joints designed for less than the strength of the member do not contribute sufficiently to member stiffness for this assumption and deflection calculations, as well as recalculations, of moment will require the assumption of variable cross-sections through the joint itself.

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